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**OPERATIONALLY - RELEVANT TEST LENGTHS:
A DECISION - ANALYSIS APPROACH**

by

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OPERATIONALLY - RELEVANT TEST LENGTHS: A DECISION - ANALYSIS APPROACH

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THESIS DISCLAIMER

The reader is cautioned that models and computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the models provide accurate results and the programs are free of computational and logic errors, they must be further validated and verified. The completion of these tasks is left for further research. Any application of these models and programs without additional validation and verification is at the risk of the user.

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The author would like acknowledge the fact that the idea and structure for this model was primarily developed by Distinguished Professor Donald Gaver. Professor Patricia Jacobs contributed greatly in her verification of the numbers produced by my algorithm. I wish to express my sincere thanks to them both for their time and patience.

EXECUTIVE SUMMARY

As a result of the end of the Cold War, the size and structure of the United States military continues to undergo serious scrutiny. Although this is a dynamic process, the exact outcome of which is unknown, it is clear that in the future there will be significant reductions and changes in force structure, and a subsequent decrease in requirements for the generation of large numbers of new weapon systems.

This trend will impact Testing and Evaluation programs, in that fewer dollars will be allocated for the test and evaluation infrastructure and process. As with other aspects of the acquisition process, the amount of testing performed should be determined by a cost-effectiveness measure. This can be described as a balance between the expense of testing and the reduction in risk for fielding the system gained by testing. One of the more expensive components of weapon system testing is that which involves destructive testing such as firing missiles or detonating ordinance. Such actions are required to provide some verification of the systems' operational effectiveness and reliability. The need for testing in an operational environment adds further to the expense and complexity of the system evaluation. Clearly, testing methods which balance the need for economical efficiency and statistical integrity must be explored in order for the war fighter to continue to receive systems that are both effective and suitable, with firm attention to cost and value added.

This thesis presents a formal analytical process for arriving at a number, t*, of missiles (or other expendable items) to test out of a finite lot or 'buy" of m. It does so by estimating the operational combat utility of the missile, given acceptance. This means that testing focuses on adding operational value, rather than on simply reducing uncertainty, as simple hypothesis testing procedures tend to do. The methodology accommodates two different tactical usages for the missile: a single shot, or a salvo of two shots. The missile might be acceptable if used in the two-shot salvo mode, but not in the single shot mode, and this would imply a greater cost per mission. In the end the missile might not be judged cost-effective as compared to a competitive system. If the model proposed is (or

can become) adequate much can be calculated/estimated before any operational tests are made. This could assist in economizing on operational testing.

The method proposed here is a suggestion and an approach; it is not a finished product. Various questions must be answered before the approach is practical. For example: how does one specify the prior (parameters) and the probability of successful opponent retaliation? Answer: from organizational experience with analogous systems, and from distilled expert judgment. Also, what to do with a system that is rejected (after testing an appropriate number of times and getting fewer than minimum number of successes)? The model does not attempt to address the choice of whether to end the program, or to look for particular faults that caused the deficiency and attempt to correct them. This choice is situation-specific, but if the system capability is needed and the faults are identifiable and rectifiable at reasonable cost then the latter course is attractive. Careful retrospective analysis of the test conditions is always important, whatever the outcome.

In summary it is argued that some organized and defensible test planning and decision aiding process is needed by the Operational Testing community. The present approach is a proposed step on the path to filling that need.

I. INTRODUCTION

A. BACKGROUND

As a result of the end of the Cold War, the size and structure of the United States military continues to undergo serious scrutiny. Although this is a dynamic process, the exact outcome of which is unknown, it is clear that in the future there will be significant reductions and changes in force structure, and a subsequent decrease in requirements for the generation of large numbers of new weapon systems.

This trend will impact Testing and Evaluation programs, in that fewer dollars will be allocated for the test and evaluation infrastructure and process. As with other aspects of the acquisition process, the amount of testing performed should be determined by a cost-effectiveness measure. This can be described as a balance between the expense of testing and the reduction in risk for fielding the system gained by testing. One of the more expensive components of weapon system testing is that which involves destructive testing such as firing missiles or detonating ordinance. Such actions are required to provide some verification of the systems' operational effectiveness and reliability. The need for testing in an operational environment adds further to the expense and complexity of the system evaluation. Clearly, testing methods which balance the need for economical efficiency and statistical integrity must be explored in order for the war fighter to continue to receive systems that are both effective and suitable, with firm attention to cost and value added.

B. CURRENT THINKING CONCERNING UNCERTAINTY

When a test is designed it is set up to answer a decision maker's question such as, "How good is this?" or "Is it better than the one we have?" The challenge for the test designer is to translate these questions into something that is testable. Typically this translation results in statistical hypothesis testing. A key consideration in this type of testing is the fact that the statistician will never tell you that the hypothesis has been proven correct. He will only tell you whether or not it has been rejected. That is to say, statistical hypothesis testing can lead to only two outcomes: the hypothesis is rejected, or

the hypothesis cannot be rejected. Statistical hypothesis testing never directly confirms the hypothesis being tested. It is evident that the formulation or wording of the hypothesis is critical to the information the testing produces. Furthermore, conventionally applied hypothesis testing does not take into account gains from making correct decisions, or losses from making incorrect ones.

Statistical hypothesis testing has another unattractive quality in these days of shrinking budgets in that it does not allow the tester to take advantage of prior knowledge. Only the data gathered in the context of the particular test formally qualifies for analysis. Information available from other resources such as developmental testing, testing of similar systems, field experience and the like are formally disregarded. The reason this could be unattractive in today's fiscal environment is obvious. Information gained from another similar system's testing process may remove the need for at least a portion of testing of the new system and thereby conserve the relevant funds.

C. ALTERNATIVE THINKING

Bayes' theorem gives a probabilistically-based rule for updating the degree of belief in a hypothesis H (i.e., the probability of H) given additional evidence E and background information (context) I:

$$P\{H|E,I\} = \frac{P\{H|I\}P\{E|H,I\}}{P\{E|I\}}. \tag{1.1}$$

Note that all the probabilities are conditional. They specify the degree of our belief in some proposition under the assumption that some other propositions are true. This would seem to imply that Bayesian methods require the addition of more assumptions to obtain results, which is true. The most important assumption made is that of the value of the "prior," the $P\{H|I\}$ term in the equation above. Here lies the rub. Prior probabilities have been not only ignored but abhorred by some classical statisticians. This has been due chiefly to the frequently subjective nature of the prior probabilities. In practice there may well be sufficient domain knowledge to specify a prior. In general, prior probabilities can be assigned to any unknown parameters involved in a formulation. After additional data is

obtained these priors can be updated to a posterior. Clearly, Bayesian methods can take advantage of knowledge from other sources that is relevant to the testing, and thereby potentially save cost and time during the test and evaluation phase of system acquisition.

D. THESIS OBJECTIVE

This thesis employs a Bayesian framework in order to address a simplified version of a decision problem in destructive testing. It approaches the question of "How much testing is enough?" by formulating a model for the combat situation in which the weapon (e.g., missile) will be used. It then employs sequential Bayesian thinking to infer the testing level needed to achieve optimal expected gain. Questions of risk are also addressed.

II. PROBLEM FORMULATION AND SOLUTIONS

A. INTRODUCTION

A new system is proposed for acquisition, possibly to replace or upgrade an existing system. This new system will be accepted provided it meets minimum requirements for effectiveness and suitability which will be demonstrated through testing. This testing will be performed to replicate the mission and environment to which the system is expected to be exposed, and the results of a single test will be binary: success or failure. For the purposes of this discussion we assume the system is a missile and that the testing is destructive. That is to say, the test will destroy the missile but the record of its performance will be retained.

We begin with m missiles which arrive as a lot and some number, t ($0 \leq t \leq m$), of them which are to be tested to determine whether or not the lot will be accepted. An optimization problem presents itself. If a small number of missiles (2 for example) are tested, less information is gained but a larger number of missiles is available to the fleet for use, given that the lot is accepted. Conversely, if a large number of missiles (m-2 for example) is tested, much more information is gained but only a few of the weapons are available for the missions for which they were designed (in this case, only 2). The intention of this paper is to provide a formulation that links the information gained from testing to the effect that gathering that information has on the missile's future performance. In addition, the possibility of informing the decision as to how the missile could be "best" employed, or the choice of a shooting policy, will be discussed.

B. UNCERTAINTY AND OPERATIONAL GAIN

Assume that each of the missiles has the same, constant, and unknown probability of success, p. The decision as to whether or not the missiles should be deployed to the fleet will be based on the value of p. An option is to take t of the m missiles and test them. The number of tests could range from 0 to m; the latter would of course result in perfect knowledge for the lot of missiles but would leave no weapons for the fleet to employ.

Models can be built to represent the missile as it would be used operationally. For example: when the weapon is launched at an opponent he may, if he is missed, return fire with a weapon of his own. Let v_w be the value of a win, meaning a weapon hit *on* the enemy and v_l be the value of a loss, a weapon hit *by* the enemy; q is the probability that the enemy will have a successful counterfire given missile miss. For simplicity, assume the value of q is known. The resulting gain function is, after t tests,

$$G_1(p,t;m) = (m-t)\left[v_w p - v_l(1-p)q\right] \tag{2.1}$$

and represents the total expected gain from fielding a missile with success probability p determined from t tests which leaves (m-t) possible engagements. The subscript on G_1 indicates that one weapon at a time was fired at the enemy. A second model, that describes the policy of firing a 2-weapon salvo, has gain function

$$G_2(p,t;m) = \left\lfloor \frac{m-t}{2} \right\rfloor \left[v_w\left(1-(1-p)^2\right) - v_l(1-p)^2 q\right] \tag{2.2}$$

$$\approx \frac{m-t}{2}\left[v_w\left(1-(1-p)^2\right) - v_l(1-p)^2 q\right]. \tag{2.2a}$$

Where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. We use the approximation in the thesis. Note that a shoot-look-shoot policy is <u>not</u> evaluated here. The latter, while possibly more economical of fielded missile inventory, may well increase the chance of successful counterfire by the opponent (a larger q value).

C. THE DECISION RULE

1. Probability of Success Known

If the decision maker knows the value of p he might then opt not to test, meaning that t=0. He would then evaluate $G_i(p, 0; m)$: if the result is sufficiently positive, (exceeds a gain threshold), he accepts the system and fields it, while if the result is not sufficiently positive he rejects the system, achieving a gain of zero. The acceptance gain threshold may be defined by a predetermined minimum value of gain or by a minimum improvement in gain relative to a preceding system. This use of gain functions is of course quite simplified but serves the purpose of illustrating the method and provoking further

thought. In any case the decision whether or not to accept a system reduces to whether or not the system's probability of success, p, meets or exceeds some minimum threshold, $\underline{p}$. For example, in the case of the gain function for G_1, it can be seen that in order for the system to have positive gain, we must have a p-value that exceeds $\underline{p} = v_1 q / (v_w + v_1 q)$.

2. Uncertainty in p: Bayesian Approach to Acceptance, Given a Test

The decision to test t leads to the acquisition of data which is assumed to be summarized entirely as s successes $(s = 0, 1, 2, \ldots, t)$. These data can now be used in the development of a likelihood for p by use of a binomial model, and, if p has a beta prior, a beta posterior for p:

$$\Pi(p; s, t) = B(\alpha', \beta') p^{\alpha'-1} (1-p)^{\beta'-1} \tag{2.3}$$

where $\alpha' = \alpha + s$ and $\beta' = \beta + t - s$. The values (α, β) characterize the prior density for **p**. This is the classical conjugate prior setup which is standard; see Appendix A for details of the derivation. If more investigation shows that an alternative prior is more appropriate it is straightforward to supply the necessary changes. Now a decision maker in possession of the posterior, equation (2.3), should use it to evaluate the expected gain. This is accomplished by computing the gain's expected value with respect to the posterior probability distribution, equation (2.3). In the case of G_1, the expectation is linear and of the form,

$$E\left[G_1(\mathbf{p}, t; m) \middle| s, t\right] = (m - t)\left[v_w E\left[\mathbf{p} \middle| s, t\right] - v_1\left(1 - E\left[\mathbf{p} \middle| s, t\right]\right) q\right], \tag{2.4}$$

representing the expected gain from a fielded system. An appropriate decision rule for this case is to field the system if $E\left[G_1(\mathbf{p}, t; m) \middle| s, t\right]$ is positive; otherwise "reject" the system. This is mathematically equivalent to fielding the system if $E\left[G_1(\mathbf{p}, t; m) \middle| s, t\right] \geq \underline{g}$, where $\underline{g}$ might represent the gain from utilizing an alternative system. Because of the form of G_1 this is also equivalent to fielding the system if $s(t) \geq \underline{s}(t)$ where $\underline{s}(t)$ represents the minimum number of successes out of t tests which results in a positive value for G_1 (or a

value of $\underline{g} > 0$). The derivation of $\underline{s}(t)$ for both the single-weapon and two-weapon salvo cases can be found in Appendix B.

D. HOW MUCH TO TEST

1. Single-Weapon Case

The previous section indicates what decision to make given the test extent, t, and the number of resultant successes, s(t). Now take the position of the decision maker before any destructive testing is performed. He must consider testing to any level. That is to say, he must consider testing 1, 2, 3, up to m missiles and determining which of the values for t provides the best potential for gain from the remaining missiles. Of course the results of the testing must satisfy the decision rules previously stated for the system to be fielded. The prediction used depends on the binomial model and upon the prior, which is assumed to be the beta prior with parameters α and β:

$$\Pi(p) = B(\alpha,\beta)p^{\alpha-1}(1-p)^{\beta-1}. \tag{2.5}$$

Conditionally upon p,

$$P\{s(t) = s|\mathbf{p},t\} = \binom{t}{s}\mathbf{p}^{s}(1-\mathbf{p})^{t-s}, \qquad 0 \le s \le t. \tag{2.6}$$

In order to predict s(t) simply remove the condition on p, using the prior; this encapsulates the decision maker's uncertain knowledge at the time he must decide on t. Details are found in Appendix A. The resultant predictive distribution, b(s;t), is the beta-binomial

$$b(s;t) \equiv P\{s(t) = s|t\} = \binom{t}{s}\left(\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right)\left(\frac{\Gamma(\alpha+s)\Gamma(\beta+t-s)}{\Gamma(\alpha+\beta+t)}\right). \tag{2.7}$$

Now the expectation of gain, $E[G_1]$, depends on the value of s(t), which is unknown during the planning phase of the test but whose probability distribution is given by equation (2.7), a beta-binomial. Consequently, the value of future gain can be predicted by unconditioning on s:

$$E\left\{E\left[G_1(\mathbf{p},t;m)\middle|s,t\right]\right\} = (m-t)\left[(v_w + v_l q)\sum_{x \geq \underline{s}(t)} x b(x;t) - v_l q \sum_{x \geq \underline{s}(t)} b(x;t)\right]. \tag{2.8}$$

This expected gain can be evaluated over the entire range of t in principle.

2. Two-Weapon Salvo Case

Maintaining the aforementioned position of the decision maker and the questions facing him, we address the case in which 2 weapons are fired sequentially with no delay between shots. The beta prior, equation (2.5) and predictive distribution for **s**(t), equation (2.7), remain unaltered from the single weapon case. Thus, the expectation of G_2 also depends on the value of **s**(t) and can be predicted by calculating

$$E\left\{E\left[G_2(\mathbf{p},t;m)\middle|\mathbf{s}(t),t\right]\right\} = \left\lfloor \frac{m-t}{2} \right\rfloor \left[v_w \sum_{x \geq \underline{s}(t)} b(x;t) - (v_w + v_l q) \sum_{x \geq \underline{s}(t)} \left(\frac{(\beta+t-x+1)(\beta+t-x)}{(\alpha+\beta+t+1)(\alpha+\beta+t)}\right) b(x;t)\right]$$

(2.9)

The derivation of $\underline{s}(t)$ can be found in Appendix B. This expected gain can also be evaluated over the entire range of t and consideration should be given to starting with small values of t and proceeding as in the single weapon case.

E. RISK OF ACCEPTANCE

The value of t, denoted by t*, which results in the maximum value of the predicted gain is determined by numerical search as described previously. Given that during the testing at least $\underline{s}(t)$ successes were observed and that the system was subsequently accepted, we are now interested in the <u>risk</u> associated with this acceptance. Two possible measures of this risk are (a) the predicted probability of kill per engagement (e.g., a successful engagement), and (b) the probability that the number of mission engagements result in fewer successes than some threshold, D. Such information supplements that given by the expected value of gain, which involves not just **p** but also the chosen costs or penalties v_w, v_l, and q.

1. Single-Weapon Case

The predicted probability of kill per engagement given acceptance of the system emerges from the now familiar beta-binomial distribution, conditional on acceptance of the system, $s(t) \geq \underline{s}(t)$ and with the beta based on the prior parameters, α and β:

$$\bar{p}(t) = \frac{\sum\limits_{s \geq \underline{s}(t)} \left(\frac{\alpha+s}{\alpha+\beta+t}\right) b(s;t)}{\sum\limits_{s \geq \underline{s}(t)} b(s;t)}. \tag{2.10}$$

Let **M**(t) be the number of successful engagements from utilizing the m-t missiles in combat. The probability that **M**(t) will fall short of some minimum standard, D, for kills can be found in terms of a binomial. First,

$$P\{\mathbf{M}(t) = k | p, s\} = \binom{m-t}{k} p^k (1-p)^{m-t-k}; \quad k = 0,1,2,\ldots,m-t. \tag{2.11}$$

Next, remove the condition on p using the beta posterior:

$$P\{\mathbf{M}(t) = k | s\} = \binom{m-t}{k} \frac{\Gamma(\alpha+\beta+t)}{\Gamma(\alpha+s)\Gamma(\beta+t-s)} \frac{\Gamma(\alpha+s+k)\Gamma(\beta-s+m-k)}{\Gamma(\alpha+\beta+m)}, \tag{2.12}$$

and the condition on s, given acceptance, yields after simplification,

$$P\{\mathbf{M}(t) = k | \text{Acceptance}\} = \frac{\binom{m-t}{k}\Gamma(\alpha+\beta+t)}{\Gamma(\alpha+\beta+m)} \frac{\sum\limits_{j \geq \underline{s}(t)} \binom{t}{j} \Gamma(\alpha+j+k)\Gamma(\beta-j+m-k)}{\sum\limits_{j \geq \underline{s}(t)} \binom{t}{j} \Gamma(\alpha+j)\Gamma(\beta+t-j)}. \tag{2.13}$$

Finally,

$$P\{\mathbf{M}(t) \leq D | \text{Acceptance}\} = \sum_{k=0}^{D} P\{\mathbf{M}(t) = k | \text{Acceptance}\}. \tag{2.14}$$

Now the above, (2.14), can be evaluated at $t = t^*$ for various D-values. It may be judged that the probability of mission success is simply intolerably low despite the fact that expected gain is positive. In that case the acceptance criterion, $\underline{s}(t)$, may be adjusted, and attempts made to increase the intrinsic capability of the new missile system, which could encompass performance as well as tactical considerations.

2. Two-Weapon Salvo Case

In this case the probability of kill per engagement is found by first deriving the expectation of the probability that both weapons fired in the salvo miss their intended target, $(1-p)^2$. This expected probability of failure is found in a similar fashion to that of the expected probability of success using the beta posterior, described in the single-weapon case. The expected posterior probability of success is then $E\left[1-(1-\mathbf{p})^2\right]$. Details are available in Appendix A. The predicted probability of kill per engagement, like the single-weapon case, is conditional on acceptance of the system, $s(t) \geq \underline{s}(t)$ and with the prior distribution with parameters, α and β:

$$\bar{p}(t) = \frac{\sum_{s \geq \underline{s}(t)} \left(1 - \frac{(\beta+t-s+1)(\beta+t-s)}{(\alpha+\beta+t+1)(\alpha+\beta+t)}\right) b(s;t)}{\sum_{s \geq \underline{s}(t)} b(s;t)}. \tag{2.15}$$

Let $\mathbf{M}(t)$ be the number of successful engagements from utilizing the m-t missiles in combat. Since the missiles will be expended in pairs, let N(m,t) represent the maximum number of possible engagements with the missiles that remain after testing which is given by the largest integer less than or equal to $\frac{m-t}{2}$ denoted by $\left\lfloor \frac{m-t}{2} \right\rfloor$. The probability that $\mathbf{M}(t)$ will fall short of some minimum standard, D, for kills can also be found in terms of a binomial:

$$P\{M(t) = k | p, s(t)\} = \binom{N(m,t)}{k} \left(1-(1-p)^2\right)^k \left((1-p)^2\right)^{N(t,m)-k}; \; k = 0,1,2,\ldots,N(m,t). \tag{2.16}$$

Removing the condition on $\mathbf{p}$ using the beta posterior (2.7) and the binomial theorem (See Appendix C),

$$P\{\mathbf{M}(t) = k | s\} =$$

$$\binom{N(m,t)}{k} \sum_{n=0}^{k} \binom{k}{n} (-1)^n \frac{\Gamma(\alpha+\beta+t)}{\Gamma(\beta+t-s)} \frac{\Gamma(2N(m,t)-2k+2n+\beta+t-s)}{\Gamma(2N(m,t)-2k+2n+\alpha+\beta+t)}, \tag{2.17}$$

and the condition on s yields after simplification,

$$P\{\mathbf{M}(t) = k | \text{Acceptance}\} =$$

$$\frac{\displaystyle\sum_{j \ge \underline{s}(t)} \sum_{n=0}^{k} \binom{t}{j} \binom{N(m,t)}{k} \binom{k}{n} (-1)^n \frac{\Gamma(\alpha+j)\Gamma(2N(m,t)-2k+2n+\beta+t-j)}{\Gamma(2N(m,t)-2k+2n+\alpha+\beta+t)}}{\displaystyle\sum_{j \ge \underline{s}(t)} \binom{t}{j} \frac{\Gamma(\alpha+j)\Gamma(\beta+t-j)}{\Gamma(\alpha+\beta+t)}}, \quad k = 0,1,2,\ldots N(m,t). \tag{2.18}$$

Finally,

$$P\{\mathbf{M}(t) \le D | \text{Acceptance}\} = \sum_{k=0}^{D} P\{\mathbf{M}(t) = k | \text{Acceptance}\}. \tag{2.19}$$

As with the single-weapon case, the above, (2.19), can be evaluated at $t = t^*$ for various D-values. It may be judged that the probability of mission success is simply unacceptably low despite the fact that expected gain is positive. In that case the acceptance criterion, $\underline{s}(t)$ may be adjusted, and attempts made to increase the intrinsic capability of the new missile system, which could encompass performance as well as tactical considerations.

III. NUMERICAL EXAMPLE

A. INTRODUCTION

Consider a lot of 30 missiles offered by a manufacturer who has performed developmental testing and is satisfied that the uncertainty in each missile's kill probability per engagement can be described by a beta distribution with parameters α and β, equal to 5 and 1 respectively. We assume (2.1) adequately models the system in its combat environment and that an expected gain of 0 or greater is sufficient for acceptance although a gain of 0 might indicate no improvement over a current system. We further assume the values for v_w, v_l, and q are known to be 1, 5, and 0.75. The 1 and 5 serve to indicate that the (negative) value of losing one of our aircraft is 5 times that of causing the destruction of one of the opponent's aircraft. The 0.75 indicates that if the enemy has opportunity to counterfire he will with 0.75 probability succeed in shooting down our aircraft.

B. THE ACCEPTANCE POLICY

Accepting the system based upon a minimum expected gain is mathematically equivalent to accepting the system if $s(t) \geq \underline{s}(t)$. The value of $\underline{s}(t)$ is found by evaluating (B.6):

$$\underline{s}(t) = \frac{v_l q(\alpha + \beta + t)}{v_w + v_l q} - \alpha$$

$$= \frac{5(0.75)(5+1+t)}{1+5(0.75)} - 5$$

$$\underline{s}(t) = \tfrac{15}{19}t - \tfrac{5}{19} \tag{3.1}$$

Negative values of $\underline{s}(t)$ are set equal to 0 while all other values are rounded up to the nearest integer. This is consistent with both the idea that only non-negative numbers of successes can be observed and that $\underline{s}(t)$ is to provide a lower bound on acceptable gain which is itself restricted to values greater than or equal to 0.

Next $\underline{s}(t)$ and (2.7) are used to evaluate the predicted expected gain given by (2.8) for each value of t $(0 \le t \le 30)$.

$$E\{E[G_1(\mathbf{p},t;m)|s(t),t]|\text{Acceptance}\} = (m-t)\left[(v_w + v_1 q)\sum_{x \ge \underline{s}(t)} x b(x;t) - v_1 q \sum_{x \ge \underline{s}(t)} b(x;t)\right]$$

$$= (30-t)\left[(4.75)\sum_{x \ge \underline{s}(t)} 5x\binom{t}{x}\frac{\Gamma(5+x)\Gamma(1+t+x)}{\Gamma(6+t)} - 3.75\sum_{x \ge \underline{s}(t)} 5\binom{t}{x}\frac{\Gamma(5+x)\Gamma(1+t+x)}{\Gamma(6+t)}\right] \tag{3.2}$$

Relevant values of t, $\underline{s}(t)$, and predicted gain can be seen in Table 3.1. If the predicted gain were determined to be zero through the range of t, the system would be rejected without testing since there is at best no gain from the testing expenditure. Also, if the maximum expected gain is achieved with no testing, t = 0, then the evaluation of the risk

t	0	1	2	3	4	5	6	7
$\underline{s}$(t)	0	1	2	3	3	4	5	6
E[G_1]	6.25	7.77	8.13	7.97	7.94	8.05	7.95	7.72

Table 3.1. Example Results.

associated with deployment of the system becomes more significant. Neither of these situations occur in this example. In this example maximum expected gain occurs when 2 of the 30 missiles are tested and the remaining 28 missiles are deployed. The policy is to accept the missile if both of the tests are successful. It is noticeable that the expected gain has a somewhat flat or slowly-changing dependence on t, so a judgment that a few more tests are desirable, based on unmodeled considerations, does not incur a great change in expected gain. Extra tests might be conducted to reveal surprising system faults. Additional testing under different tactical conditions may very well be necessary.

C. RISK ASSESSMENT

We are now interested in the risk associated with acceptance of the missile system subsequent to observing at least $\underline{s}(t)$ successes during testing. The first measure of risk

considered is the predicted probability of kill per engagement given acceptance which can be obtained from (2.10) in concert with (2.7). The same numerical example is used for illustration.

$$\bar{p}(t) = \frac{\sum\limits_{x \geq s(t)} \left(\frac{\alpha+x}{\alpha+\beta+t}\right) b(x;t)}{\sum\limits_{x \geq s(t)} b(x;t)} = \frac{\sum\limits_{x=2}^{2} \frac{5+x}{8} \binom{2}{x} \frac{\Gamma(6)}{\Gamma(5)} \frac{\Gamma(5+x)\Gamma(3+x)}{\Gamma(8)}}{\sum\limits_{x=2}^{2} \binom{2}{x} \frac{\Gamma(6)}{\Gamma(5)} \frac{\Gamma(5+x)\Gamma(3+x)}{\Gamma(8)}} = 0.875 \qquad (3.3)$$

The second measure of risk considers the probability that the number of mission engagements result in fewer successes than some threshold, D given acceptance. This step is accomplished by evaluating (2.13)

$$P\{\mathbf{M}(t) = k | \text{Acceptance}\} = \frac{\binom{m-t}{k} \Gamma(\alpha+\beta+t)}{\Gamma(\alpha+\beta+m)} \frac{\sum\limits_{j \geq s(t)} \binom{t}{j} \Gamma(\alpha+j+k)\Gamma(\beta-j+m-k)}{\sum\limits_{j \geq s(t)} \binom{t}{j} \Gamma(\alpha+j)\Gamma(\beta+t-j)} \qquad (3.4)$$

$$= \frac{\binom{28}{k} \Gamma(8)}{\Gamma(36)} \frac{\sum\limits_{x=2}^{2} \binom{2}{x} \Gamma(5+x+k)\Gamma(31-x-k)}{\sum\limits_{x=2}^{2} \binom{2}{x} \Gamma(5+x)\Gamma(3-x)},$$

then simply sum the results for the cumulative distribution up to D. Table 3.2 contains an

D	0	1	2	3	4	12	13	14	15	24	25	26	27	28
P{M=D}	0	0	0	0	0	.002	.004	.006	.008	.088	.109	.135	.165	.2
P{M ≤ D}	0	0	0	0	0	.007	.012	.017	.025	.391	.501	.635	.8	1

Table 3.2. Risk Summary.

illustrative portion of the results for this example. In this example there is a 50% probability of 25 or fewer successes out of 28, or a 50% chance of 25 or more successes out of 28. Such information enhances the decision maker's intuition.

D. SUMMARY

The example above demonstrates a formulation that links the information gained from testing to the effect that testing has on a particular measure of the performance of the system once fielded. Despite the fact that the example is simple, valuable information is gleaned from it. In this example some guidance is provided to an individual who might be deciding just how many of the 30 available missiles should be tested. While testing 2 of the 30 missiles may not be *the* answer, if one places some belief in the assumptions made at the beginning of the exercise it is apparent that the number to test is nearer 2 than a considerably larger number, such as 15. Furthermore, the expense has been minimal compared to that of expending weaponry under one set of tactical conditions. Additional missile testing under different conditions is needed to explore the range of possible combat environments, so it may be reassuring that a comparatively few tests are recommended for each of these explorations.

IV. SOFTWARE IMPLEMENTATION

A. INTRODUCTION

The methods developed in previous chapters are, even for the simplest cases, computationally tedious. This invites one to use a computer not only for the convenience but also for the speed. Convenience is lost to some degree when one must learn a programming language in order execute the type of calculations involved in this formulation. On the other hand, more and more people are becoming familiar with spreadsheets and their use, especially in the analytical community. Increasingly, spreadsheets available commercially are suitable for situations that once required a custom program written in a traditional programming language. One such spreadsheet is Microsoft's *Excel V7.0*. *Excel* and its accompanying macro language , *Visual Basic for Applications,* were used in the production of this project's software. The code for this application is located in Appendix D. The "readme.txt" help file and software installation directions are located in Appendix E.

B. OVERVIEW

1. Input

The user is presented with a window which prompts him for the necessary information (See Figure 4.1). The inputs for both the single-weapon and two-weapon salvo cases are handled in an identical manner. Default values are indicated with the purpose of avoiding program crashes caused by blank input fields. Once the user is satisfied with his input selection, he merely clicks the "Begin" button or presses the return key on his keyboard to initiate the computations.

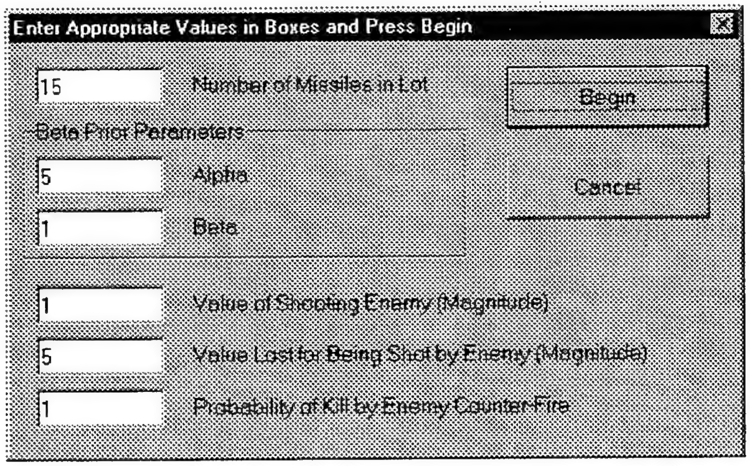
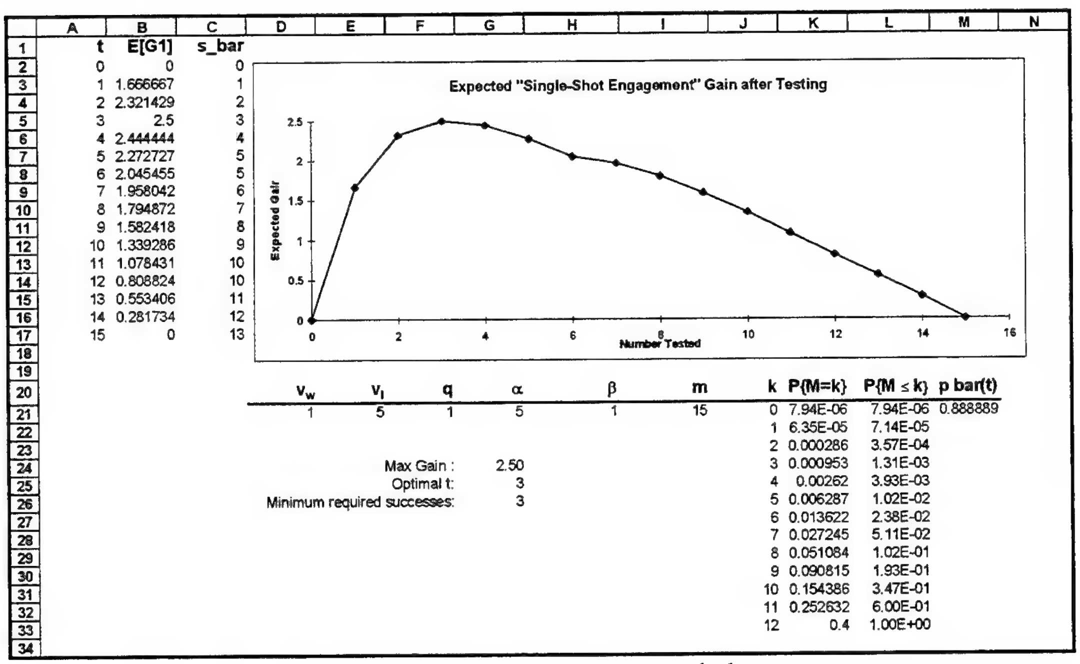


Figure 4.1. Input User Interface.

2. Output

After the user initiates the computations and the algorithm has executed, a spreadsheet page is produced. The worksheet contains a plot of the predicted gain values over the range of t (Figure 4.2) and several tables. The tables display information regarding the relationship between t, E[G_i], and $\underline{s}(t)$; the optimal number of missiles to test, t*; and the values associated with the risk incurred if the system is fielded.



t	E[G1]	s_bar
0	0	0
1	1.666667	1
2	2.321429	2
3	2.5	3
4	2.444444	4
5	2.272727	5
6	2.045455	5
7	1.958042	6
8	1.794872	7
9	1.582418	8
10	1.339286	9
11	1.078431	10
12	0.808824	10
13	0.553406	11
14	0.281734	12
15	0	13

v_w	v_l	q	α	β	m
1	5	1	5	1	15

Max Gain :	2.50
Optimal t:	3
Minimum required successes:	3

k	P{M=k}	P{M ≤ k}	p bar(t)
0	7.94E-06	7.94E-06	0.888889
1	6.35E-05	7.14E-05	
2	0.000286	3.57E-04	
3	0.000953	1.31E-03	
4	0.00262	3.93E-03	
5	0.006287	1.02E-02	
6	0.013622	2.38E-02	
7	0.027245	5.11E-02	
8	0.051084	1.02E-01	
9	0.090815	1.93E-01	
10	0.154386	3.47E-01	
11	0.252632	6.00E-01	
12	0.4	1.00E+00	

Figure 4.2. Typical Output Worksheet.

C. DEMONSTRATION

The following serve to demonstrate the scope of this project's software through various combinations of weapon cases and analysis outcomes. Although the combinations are not exhaustive, they illustrate sufficiently to express the breadth of outcome possibilities.

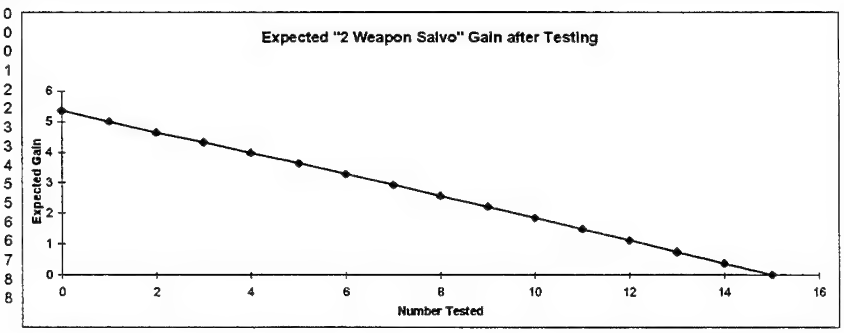
1. Single-Weapon Case with Testing Recommended

Consider a lot of 15 missiles whose prior probability of kill has a beta distribution with parameters α and β, equal to 5 and 1 respectively (Figure 4.2). The values of v_w, v_l and q are 1, 5, and 1 respectively. The values used for the other relevant parameters are displayed in the output worksheet for convenience. The algorithm finds that maximum gain is achieved when 3 missiles are tested and an acceptance policy of 3 successes from 3 trials is employed. See Figure 4.2 for these results as well as the risk of acceptance predictions.

2. Two-Weapon Salvo Case with No Testing Recommended

Consider a lot of 15 missiles whose prior probability of kill has a beta distribution with parameters α and β, equal to 5 and 1 respectively (Figure 4.3). The values of v_w, v_l and q are 1, 5, and 1 respectively. The values used for the other relevant parameters are

t	E[G2]	s_bar
0	5.357143	0
1	5	0
2	4.642857	0
3	4.321429	1
4	3.97619	2
5	3.625541	2
6	3.277972	3
7	2.914585	3
8	2.56216	4
9	2.199051	5
10	1.836399	5
11	1.472204	6
12	1.1046	6
13	0.738206	7
14	0.369306	8
15	0	8



v_w	v_l	q	α	β	m
1	5	1	5	1	15

Max Gain :	5.36
Optimal t:	0
Minimum required successes:	0

k	P{M=k}	P{M ≤ k}	p bar(t)
0	8.60E-05	8.60E-05	0.952381
1	0.000529	6.15E-04	
2	0.002012	2.63E-03	
3	0.006188	8.82E-03	
4	0.017293	2.61E-02	
5	0.047796	7.39E-02	
6	0.147272	2.21E-01	
7	0.778823	1.00E+00	

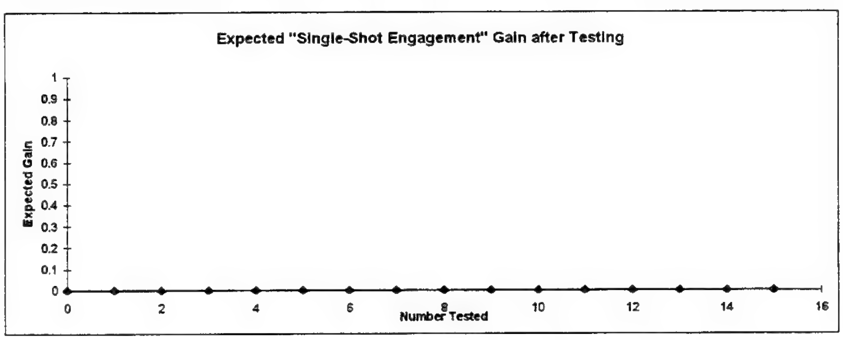
Figure 4.3. Two-Weapon Salvo Case with No Testing Recommended.

displayed in the output worksheet for convenience. Note that the cost values are the same as those in Figure 4.2. However this case envolves consideration for the two-weapon salvo where the penalty for accepting a system with a low value of p is small. Subsequently, the algorithm finds that maximum gain is achieved when 0 missiles are tested and suggests system acceptance without testing (it is likely that this would be overridden in practice). In this case the risk of acceptance analysis is particularly germane. See Figure 4.3 for these results.

3. Single-Weapon Case with System Rejection

Consider a lot of 15 missiles whose prior probability of kill has a beta distribution with parameters α and β, equal to 5 and 1 respectively (Figure 4.4). The values of v_w, v_l and q are 1, 25, and 1 respectively. The values used for the other relevant parameters are displayed in the output worksheet for convenience. The algorithm finds that maximum gain that could be achieved is no better than that gain which exists without the missile system being considered and suggests system rejection without testing. In this case the cost of loss is v_l = 25 as compared to v_l = 5 in Figure 4.3. Acceptance of this system

t	E[G1]	s_bar
0	0	
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
9	0	
10	0	
11	0	
12	0	
13	0	
14	0	
15	0	



V_w	V_l	q	α	β	m	k	P{M=k}	P{M ≤ k}	p bar(t)
1	25	1	5	1	15	REJECT	SYSTEM		

Max Gain :	0.00
Optimal t:	0
Minimum required successes:	NA

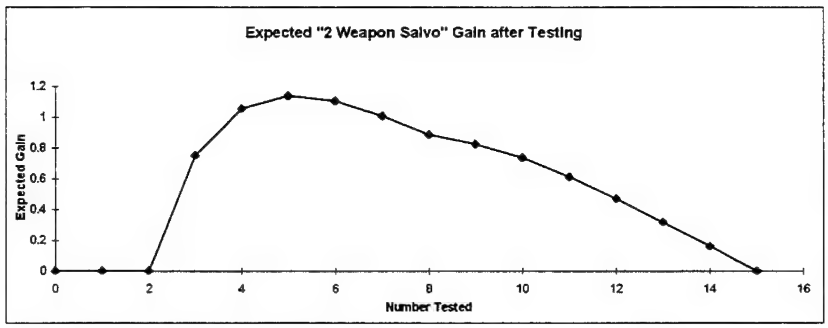
Figure 4.4. Single-Weapon Case with System Rejection.

would result in gain which is at best zero and probably negative. In other words, acceptance of the system will place the decision maker in no better a position. See Figure 4.4 for these results.

4. Two-Weapon Salvo Case with Testing Recommended

Consider a lot of 15 missiles whose probability of kill has a beta distribution with parameters α and β, equal to 5 and 1 respectively (Figure 4.5). The values of v_w, v_l and q are 1, 35, and 1 respectively. The values used for the other relevant parameters are displayed in the output worksheet for convenience. Note that testing is recommended here (accepting the system if 5 successes are observed in 5 tests) while testing is not recommended Figure 4.3. In this case the cost of a loss here is $v_l = 35$ as compared to $v_l =$

t	E[G2]	s_bar
0	0	0
1	0	0
2	0	0
3	0.75	3
4	1.055556	4
5	1.136364	5
6	1.101399	6
7	1.007326	7
8	0.884615	8
9	0.824176	8
10	0.735294	9
11	0.612745	10
12	0.470201	11
13	0.316821	12
14	0.15873	13
15	0	13



v_w	v_l	q	α	β	m
1	35	1	5	1	15

k	P{M=k}	P{M ≤ k}	p bar(t)
0	5.41E-06	5.41E-06	0.984848
1	8.72E-05	9.26E-05	
2	0.000846	9.38E-04	
3	0.006875	7.81E-03	
4	0.059094	6.69E-02	
5	0.933093	1.00E+00	

Max Gain :	1.14
Optimal t:	5
Minimum required successes:	5

Figure 4.5. Two-Weapon Salvo Case with Testing Recommended.

5 in Figure 4.3. It is reasonable that more care be taken in the present more costly case: the penalty for accepting a low value of **p** is much greater here than in the case of Figure 4.3. The algorithm finds that maximum gain is achieved when 5 missiles are tested and an acceptance policy of 5 successes from 5 trials is employed. See Figure 4.5 for these results as well as the risk of acceptance predictions.

V. SUMMARY DISCUSSION

This thesis presents a formal analytical process for arriving at a number, t*, of missiles (or other expendable items) to test out of a finite lot or 'buy" of m. It does so by estimating the operational combat utility, called gain, of the missile, given acceptance. This means that testing focuses on adding operational value, rather than on simply reducing uncertainty, as simple hypothesis testing procedures tend to do. The methodology accommodates two different tactical usages for the missile: a single shot, or a salvo of two shots. The missile might be acceptable if used in the two-shot salvo mode, but not in the single shot mode, and this would imply a greater cost per mission. In the end the missile might not be judged cost-effective as compared to a competitive system. If the model proposed is (or can become) adequate much can be calculated/estimated before any operational tests are made. This could assist in economizing on operational testing.

The method proposed here is a suggestion and an approach; it is not a finished product. Various questions must be answered before the approach is practical. For example: how does one specify the prior (parameters) and the probability of successful opponent retaliation? Answer: from organizational experience with analogous systems, and from distilled expert judgment. Also, what to do with a system that is rejected (after testing t* times and getting fewer than $\underline{s}(t^*)$ successes)? The model does not attempt to address the choice of whether to end the program, or to look for particular faults that caused the deficiency and attempt to correct them. This choice is situation-specific, but if the system capability is needed and the faults are identifiable and rectifiable at reasonable cost then the latter course is attractive. Careful retrospective analysis of the test conditions is always important, whatever the outcome.

In summary it is argued that some organized and defensible test planning and decision aiding process is needed by the OT test community. The present approach is a proposed step on the path to filling that need.

APPENDIX A. BINOMIAL BAYES

The probability of a number of successes given a fixed number of trials is calculated using the binomial distribution. The usual parameters are n, the number of trials; s, the number of successes in n trials; and p, the probability of success for any of the single trials. In the case of the beta-binomial distribution the parameter p is assumed to be random having a beta distribution. After n trials have been executed determining s, the Bayes update of the beta prior for **p** is possible.

A. THE BETA POSTERIOR

Assuming a Beta prior for **p**,

$$\Pi_o(p) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}, \quad 0 \le p \le 1; 0 < \alpha; 0 < \beta, \tag{A.1}$$

and binomial sampling (n trials, s successes), the likelihood function for p is

$$L(p; n, s) = \binom{n}{s} p^s (1-p)^{n-s}. \tag{A.2}$$

The Bayes posterior is found by multiplying the prior for **p** by the likelihood and normalizing

$$\Pi(p|n, s, \alpha, \beta) = \frac{\binom{n}{s} p^s (1-p)^{n-s} p^{\alpha-1}(1-p)^{\beta-1}}{\int_0^1 \binom{n}{s} p^s (1-p)^{n-s} p^{\alpha-1}(1-p)^{\beta-1} dp} \tag{A.3a}$$

$$\Pi(p|n, s, \alpha, \beta) = Cp^{\alpha+s-1}(1-p)^{\beta+n-s-1}. \tag{A.3b}$$

Also, since the density must integrate to 1,

$$C = \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+s)\Gamma(\beta+n-s)}. \tag{A.4}$$

Then, the beta posterior mean for **p** is

$$E[\mathbf{p}; \alpha, \beta | s, n] = \frac{\alpha+s}{\alpha+\beta+n}. \tag{A.5}$$

Later the expectation of $(1\text{-}\mathbf{p})^2$ will be needed. It can be derived in a fashion similar to that above

$$E\left[(1-\mathbf{p})^2 \middle| n,s\right] = B(\alpha',\beta')\int_0^1 (1-p)^2 p^{\alpha'-1}(1-p)^{\beta'-1}dp \tag{A.6a}$$

$$E\left[(1-\mathbf{p})^2 \middle| n,s\right] = \frac{\Gamma(\alpha'+\beta')}{\Gamma(\alpha')\Gamma(\beta')}\int_0^1 p^{\alpha'-1}(1-p)^{\beta'+2-1}dp \tag{A.6b}$$

where $\alpha' = \alpha + s$ and $\beta' = \beta + t - s$. Finally, the mean beta posterior is

$$E\left[(1-\mathbf{p})^2 \middle| n,s\right] = \frac{(\beta+n-s+1)(\beta+n-s)}{(\alpha+\beta+n+1)(\alpha+\beta+n)}. \tag{A.7}$$

B. THE PREDICTIVE DISTRIBUTION FOR THE NUMBER OF SUCCESSES

This derivation takes advantage of the beta prior for **p** in the development of a predictive distribution for the number of successes, s, in t trials. A binomial model that utilizes the beta prior, (A.1), and that is conditional on p is formed

$$P\{s(t) = s|p,t\} = \binom{t}{s} p^s (1-p)^{t-s}. \tag{A.8}$$

Removing the condition on **p** yields

$$P\{s(t) = s|t\} = \int_0^1 \binom{t}{s} p^s (1-p)^{t-s} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}dp. \tag{A.9}$$

Let $\alpha' = \alpha + s$ and $\beta' = \beta + t - s$ and combine terms to obtain

$$P\{s(t) = s|t\} = \binom{t}{s} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 p^{\alpha'-1}(1-p)^{\beta'-1}dp \tag{A.10}$$

and simplify to get

$$P\{s(t) = s|t\} = \binom{t}{s} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+s)\Gamma(\beta+t-s)}{\Gamma(\alpha+\beta+t)}. \tag{A.11}$$

APPENDIX B. DERIVATION OF MINIMUM SUCCESS THRESHOLD

This appendix describes and then derives an expression for $\underline{s}(t)$, the minimum number of successes needed during testing in order to accept the system being tested.

A. INTRODUCTION

Consider a decision policy in which a system must provide a minimum level of gain from testing and subsequent fielding, given that the system is accepted. Let this minimum gain be $\underline{g}$. Because of the form of the gain functions, (2.1) and (2.2), used in this paper acceptance of the system based on observing at least a minimum number of successes during testing, $\underline{s}(t)$, is mathematically equivalent to acceptance of the system based on receiving a minimum gain, $\underline{g}$.

B. DERIVATION OF $\underline{s}(t)$

1. Single-Weapon Case

For ease of discussion, a gain of 0 has been deemed the minimum value of gain which allows system acceptance in this paper,

$$E\left[G_1\left(\mathbf{p}, t; m \middle| s(t), t\right)\right] \geq 0. \tag{B.1}$$

From (2.4) it is evident that

$$(m-t)\left[v_w E\left[\mathbf{p} \middle| s(t), t\right] - v_1 q\left(1 - E\left[\mathbf{p} \middle| s(t), t\right]\right)\right] \geq 0 \tag{B.2}$$

implies that

$$E\left[\mathbf{p} \middle| s(t), t\right] \geq \frac{v_1 q}{v_w + v_1 q}. \tag{B.3}$$

Next substituting the beta posterior (A.5),

$$\frac{\alpha + s(t)}{\alpha + \beta + t} \geq \frac{v_1 q}{v_w + v_1 q} \tag{B.4}$$

and solving for s(t) gives

$$s(t) \geq \frac{v_1 q(\alpha+\beta+t)}{v_w + v_1 q} - \alpha. \tag{B.5}$$

Since $s(t) \geq \underline{s}(t)$ for non-negative gain it follows that

$$\underline{s}(t) = \left\lceil \frac{v_1 q(\alpha+\beta+t)}{v_w + v_1 q} - \alpha \right\rceil \tag{B.6}$$

where $\lceil z \rceil$ is the smallest non-negative integer greater than or equal to z.

2. Two-Weapon Salvo Case

Again for ease of discussion, a gain of 0 has been deemed to be in this paper, the minimum value of gain which allows system acceptance.

$$E\left[G_2\left(\mathbf{p}, t; m \middle| s(t), t\right)\right] \geq 0. \tag{B.7}$$

In a fashion similar to the single-weapon case,

$$\frac{(m-t)}{2}\left[v_w - (v_w + v_1 q) E\left[(1-\mathbf{p})^2 \middle| \underline{s}(t), t\right]\right] \geq 0 \tag{B.8}$$

and substituting the beta posterior, (A.7), yields for the bracketed term

$$\left[v_w - (v_w + v_1 q)\frac{(\beta+t-s+1)(\beta+t-s)}{(\alpha+\beta+t+1)(\alpha+\beta+t)}\right] \geq 0. \tag{B.9}$$

(Note we are replacing $\left\lfloor \frac{m-t}{2} \right\rfloor$ in (2.2) with $\left(\frac{m-t}{2}\right)$.)

Let

$$\Delta = \frac{v_w(\alpha+\beta+t+1)(\alpha+\beta+t)}{v_w + v_1 q} \tag{B.10}$$

and let

$$r = t - s. \tag{B.11}$$

Now rearrange (B.9) to obtain

$$\begin{gathered}(\beta+r+1)(\beta+r) - \Delta = 0 \\ r^2 + (2\beta+1)r + (\beta^2 + \beta - \Delta) = 0.\end{gathered} \tag{B.12}$$

The quadratic formula is now utilized to find the roots of (B.12). The root of interest, r*, will be the one that is real, positive and largest in magnitude. The roots can be found by computing

$$r = \frac{-2\beta - 1 \pm \sqrt{(2\beta + 1)^2 - 4(\beta^2 + \beta - \Delta)}}{2}. \tag{B.13}$$

Finally,

$$\underline{s}(t) = \max\left(t - \lfloor r^* \rfloor, 0\right) \tag{B.14}$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x.

APPENDIX C. BINOMIAL THEOREM APPLICATION

This appendix states the binomial theorem and discusses its application with regard to the derivation of a formulation for risk assessment in the two-weapon salvo case.

A. THE BINOMIAL THEOREM

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r} \tag{C.1}$$

B. APPLICATION OF BINOMIAL THEOREM

Let $\mathbf{M}(t)$ be the number of successful engagements possible after t missiles have been tested and let $N(m,t) = \left\lfloor \frac{m-t}{2} \right\rfloor$ represent the largest integer less than $\frac{m-t}{2}$. The probability of k successes out of N(m, t) engagements conditional on $\mathbf{p}$, the probability of success for a single missile, and s, the number of successes during testing of t missiles is given by

$$P\{\mathbf{M}(t) = k|\mathbf{p}, s\} = \binom{N(m,t)}{k} \left(1-(1-\mathbf{p})^2\right)^k \left((1-\mathbf{p})^2\right)^{N(m,t)-k}. \tag{C.2}$$

Applying the binomial theorem yields

$$P\{\mathbf{M}(t) = k|\mathbf{p}, s\} = \binom{N(m,t)}{k} \left((1-\mathbf{p})^2\right)^{N(m,t)-k} \sum_{n=0}^{k} \binom{k}{n} (-1)^n \left((1-\mathbf{p})^2\right)^n (1)^{k-n} \tag{C.3}$$

and reduces to

$$P\{\mathbf{M}(t) = k|\mathbf{p}, s\} = \binom{N(m,t)}{k} \sum_{n=0}^{k} \binom{k}{n} (-1)^n \left((1-\mathbf{p})^2\right)^{2N(m,t)-2k+2n}. \tag{C.4}$$

Removing the condition on $\mathbf{p}$

$$P\{\mathbf{M}(t)=k|s\}=$$

$$=\int_0^1 \binom{N(m,t)}{k}\sum_{n=0}^{k}\binom{k}{n}(-1)^n\left((1-p)^2\right)^{2N(m,t)-2k+2n}\frac{\Gamma(\alpha+\beta+t)}{\Gamma(\alpha+s)\Gamma(\beta+t-s)}p^{\alpha+s-1}(1-p)^{\beta+t-s-1} \quad (C.5)$$

$$=\binom{N(m,t)}{k}\sum_{n=0}^{k}\binom{k}{n}(-1)^n\frac{\Gamma(\alpha+\beta+t)}{\Gamma(\alpha+s)\Gamma(\beta+t-s)}\frac{\Gamma(\alpha+s)\Gamma(2N(m,t)-2k+2n+\beta+t-s)}{\Gamma(2N(m,t)-2k+2n+\alpha+\beta+t-s)}.$$

APPENDIX D. VISUAL BASIC FOR APPLICATIONS CODE

This appendix contains the Visual Basic for Applications code used in the spreadsheet implementation software.

A. G1.XLS, SINGLE WEAPON GAIN WORKBOOK

```
'====================================
'  LT John R. Gorman
'  Spring 1997
'  Test Length Decision Analysis
'====================================
Sub get_input()
'
'This section gets input for vw, vl, q, a, b, & m.  The main subroutine
'is then called and executed.
'
 Dim DBoxBegin As Boolean
 DialogSheets("Input Box").EditBoxes("input_vw").Text = "1"
 DialogSheets("Input Box").EditBoxes("input_vl").Text = "5"
 DialogSheets("Input Box").EditBoxes("input_q").Text = "1"
 DialogSheets("Input Box").EditBoxes("input_a").Text = "5"
 DialogSheets("Input Box").EditBoxes("input_b").Text = "1"
 DialogSheets("Input Box").EditBoxes("input_m").Text = "15"
 DBoxBegin = DialogSheets("Input Box").Show
 If Not DBoxBegin Then Exit Sub
 Sheets("Results").Range("vw").Value = _
   DialogSheets("Input Box").EditBoxes("input_vw").Text
 Sheets("Results").Range("vl").Value = _
   DialogSheets("Input Box").EditBoxes("input_vl").Text
 Sheets("Results").Range("q").Value = _
   DialogSheets("Input Box").EditBoxes("input_q").Text
 Sheets("Results").Range("alpha").Value = _
   DialogSheets("Input Box").EditBoxes("input_a").Text
 Sheets("Results").Range("beta").Value = _
   DialogSheets("Input Box").EditBoxes("input_b").Text
 Sheets("Results").Range("m").Value = _
   DialogSheets("Input Box").EditBoxes("input_m").Text
 Call plot_gain
End Sub

Sub plot_gain()
'
' Calculate expected mean gain for single weapon shot, display
' tables and plots.
'
   Application.ScreenUpdating = False
   Dim vw As Variant
   Dim vl As Variant
   Dim q As Variant
   Dim a As Variant
```

```
    Dim b As Variant
    Dim t As Variant
    Dim s_bar As Variant
    vw = Range("vw").Value
    vl = Range("vl").Value
    a = Range("alpha").Value
    b = Range("beta").Value
    m = Range("m").Value
    q = Range("q").Value
    Sheets("Results").Select
    Range("t").Select
    Range(ActiveCell, ActiveCell.End(xlDown)).Select
    Selection.Clear
    Range("s_bar").Select
    Range(ActiveCell, ActiveCell.End(xlDown)).Select
    Selection.Clear
    Sheets("Results").Select
    Range("E_G1").Select
    Range(ActiveCell, ActiveCell.End(xlDown)).Select
    Selection.Clear
    For t = 0 To m
      Range("t").Select
      ActiveCell.Offset(t, 0).Value = t
      s_bar = Application.RoundUp( _
          Application.Max(0, ((vl * q * (a + b + t) / (vw + vl * q)) - a)), _
          0)
      EG1 = 0
      For j = s_bar To t
        b_j1 = Exp(Application.GammaLn(a + b)) * Exp(Application.GammaLn(a + j)) * _
            Exp(Application.GammaLn(b + t - j))
        b_j2 = Exp(Application.GammaLn(a)) * Exp(Application.GammaLn(b)) * _
            Exp(Application.GammaLn(a + b + t))
        b_j3 = Application.Combin(t, j)
        b_j = b_j1 / b_j2 * b_j3
        EG1_1 = ((vw + vl * q) * (a + j) / (a + b + t)) - (vl * q)
        EG1 = EG1 + (EG1_1 * b_j)
      Next j
      EG1 = Application.Max(EG1 * (m - t), 0)
      Range("E_G1").Select
      ActiveCell.Offset(t, 0).Value = EG1
      ActiveCell.Offset(t, 1).Value = s_bar
    Next t
  Dim check_range As Range
  Dim optimal_t As Integer
  Dim t_counter As Integer
  Dim optimal_gain As Variant
  Sheets("Results").Select
  Range("E_G1").Activate
  Range(ActiveCell, ActiveCell.End(xlDown)).Select
  Set check_range = Selection
  Range("max_gain").Select
  ActiveCell.Value = Application.Max(check_range)
  optimal_gain = Application.Max(check_range)
  Range("E_G1").Activate
```

```
 For t_counter = 1 To m + 1
  If ActiveCell.Value = optimal_gain Then
      optimal_t = ActiveCell.Offset(0, -1).Value
      GoTo loop_exit
      End If
  ActiveCell.Offset(1, 0).Activate
 Next t_counter
loop_exit:
 Range("t_star").Value = optimal_t
'
'This routine calculates the risks associated with accepting the
'system after testing.
'
 Dim k As Variant
 Dim PM1 As Variant
 Dim PM2 As Variant
 Dim PM3 As Variant
 Dim PM As Variant
 Dim t_star As Variant
 Dim m_t As Variant
 Dim i As Variant
 Dim p_bar As Variant
 Dim p_bar1 As Variant
 Dim p_bar2 As Variant
 t_star = Range("t_star").Value
 m_t = m - t_star
 s_bar_star = Application.Max( _
                    (Application.RoundUp((vl * q * (a + b + t_star) _
                                  / (vw + vl * q)) - a, 0)), 0)
 Range("s_bar_star").Value = s_bar_star
 p_bar = 0
 p_bar1 = 0
 p_bar2 = 0
 Sheets("Results").Select
 Range("PM").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 Range("k").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 Range("PMD").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 Range("p_bar").Select
 Selection.Clear
 If ((t_star = 0) And Not (optimal_gain = 0)) Then
     GoTo NoTest
 ElseIf (optimal_gain = 0) Then
     GoTo Reject
 Else
 For k = 0 To m_t
  PM2 = 0
  PM3 = 0
  PM1 = (Application.Combin(m_t, k) * _
```

```
      Exp(Application.GammaLn(a + b + t_star))) / _
      Exp(Application.GammaLn(a + b + m))
 For j = s_bar_star To t_star
       PM2 = PM2 + ((Application.Combin(t_star, j) * _
                 Exp(Application.GammaLn(a + j + k)) * _
                 Exp(Application.GammaLn(b - j + m - k))))
       PM3 = PM3 + ((Application.Combin(t_star, j) * _
                 Exp(Application.GammaLn(a + j)) * _
                 Exp(Application.GammaLn(b + t_star - j))))
 Next j
    PM = PM1 * (PM2 / PM3)
 Sheets("Results").Select
 Range("PM").Select
 ActiveCell.Offset(k, 0).Activate
 ActiveCell.Value = PM
Next k
For j = s_bar_star To t_star
      p_bar1 = p_bar1 + (a + j) * ((Application.Combin(t_star, j) * _
                          Exp(Application.GammaLn(a + j)) * _
                          Exp(Application.GammaLn(b + t_star - j))))
      p_bar2 = p_bar2 + ((Application.Combin(t_star, j) * _
                   Exp(Application.GammaLn(a + j)) * _
                   Exp(Application.GammaLn(b + t_star - j))))
Next j
p_bar = p_bar1 / ((a + b + t_star) * p_bar2)
Range("p_bar").Select
ActiveCell.Value = p_bar
Range("p_bar_comment").Select
ActiveCell.Clear
Range("PMD").Select
ActiveCell.FormulaR1C1 = "=RC[-1]"
For i = 1 To m_t
   ActiveCell.Offset(1, 0).Activate
   ActiveCell.FormulaR1C1 = "=RC[-1]+R[-1]C"
Next i
Range("k").Select
For i = 0 To m_t
   ActiveCell.Value = i
   ActiveCell.Offset(1, 0).Activate
Next i
End If
GoTo Fini
Reject:
 MsgBox "Reject System"
 Range("s_bar").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 Range("s_bar_star").Value = "NA"
 Sheets("Results").Select
 Range("p_bar").Select
 Selection.Clear
 Range("p_bar_comment").Select
 Selection.Clear
 Range("PM").Select
```

```
 ActiveCell.Value = "REJECT"
 ActiveCell.Offset(0, 1).Activate
 ActiveCell.Value = "SYSTEM"
 GoTo Fini
NoTest:
 MsgBox "Optimal Gain Achieved with No Testing"
 Sheets("Results").Select
 Range("p_bar").Select
 Selection.Clear
 ActiveCell.Value = a / (a + b)
 ActiveCell.Offset(0, 1).Value = "<-- Prior probability used since 0 tested"
 Range("k").Select
 ActiveCell.Value = "NO"
 ActiveCell.Offset(0, 1).Activate
 ActiveCell.Value = "TESTING"
 ActiveCell.Offset(0, 1).Activate
 ActiveCell.Value = "DONE"
Fini:
 Sheets("Results").Range("c2").Select
End Sub
'
'This routine provides for the functions that occur on workbook opening.
'
Sub Auto_open()
 Call get_input
 ActiveWorkbook.Protect contents = True
End Sub
```

B. G2.XLS, TWO-WEAPON SALVO GAIN WORKBOOK

```
'====================================
'  LT John R. Gorman
'  Spring 1997
'  Test Length Decision Analysis
'====================================
Sub get_input()
'
'This section gets input for vw, vl, q, a, b, & m.  The main subroutine
'is then called and executed.
'
 Dim DBoxBegin As Boolean
 DialogSheets("Input Box").EditBoxes("input_vw").Text = "1"
 DialogSheets("Input Box").EditBoxes("input_vl").Text = "15"
 DialogSheets("Input Box").EditBoxes("input_q").Text = "1"
 DialogSheets("Input Box").EditBoxes("input_a").Text = "5"
 DialogSheets("Input Box").EditBoxes("input_b").Text = "1"
 DialogSheets("Input Box").EditBoxes("input_m").Text = "15"
 DBoxBegin = DialogSheets("Input Box").Show
 If Not DBoxBegin Then Exit Sub
 Sheets("Results").Range("vw").Value = _
   DialogSheets("Input Box").EditBoxes("input_vw").Text
 Sheets("Results").Range("vl").Value = _
```

```
  DialogSheets("Input Box").EditBoxes("input_vl").Text
 Sheets("Results").Range("q").Value = _
  DialogSheets("Input Box").EditBoxes("input_q").Text
 Sheets("Results").Range("alpha").Value = _
  DialogSheets("Input Box").EditBoxes("input_a").Text
 Sheets("Results").Range("beta").Value = _
  DialogSheets("Input Box").EditBoxes("input_b").Text
 Sheets("Results").Range("m").Value = _
  DialogSheets("Input Box").EditBoxes("input_m").Text
 Call plot_gain
End Sub

Sub plot_gain()
'
' Calculate expected mean gain for single weapon shot, display
' tables and plots.
'
 Application.ScreenUpdating = False
 Dim r1 As Variant
 Dim r2 As Variant
 Dim r As Variant
 Dim vw As Variant
 Dim vl As Variant
 Dim q As Variant
 Dim a As Variant
 Dim b As Variant
 Dim t As Variant
 Dim s_bar As Variant
 vw = Range("vw").Value
 vl = Range("vl").Value
 a = Range("alpha").Value
 b = Range("beta").Value
 m = Range("m").Value
 q = Range("q").Value
 Sheets("Results").Select
 Range("s_bar").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 Range("t").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 Range("E_G2").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 For t = 0 To m
   Range("t").Select
   ActiveCell.Offset(t, 0).Value = t
   r1 = -b - 0.5 + 0.5 * (1 + ((4 * vw * (a + b + t + 1) * (a + b + t)) / (vw + vl * q))) ^ 0.5
   r2 = -b - 0.5 - 0.5 * (1 + ((4 * vw * (a + b + t + 1) * (a + b + t)) / (vw + vl * q))) ^ 0.5
   If ((r1 <= 0) And (r2 <= 0)) Then GoTo Reject_t
   If (((r2 > 0) And (r1 > 0)) And (r1 > r2)) Then r = r1
   If (((r2 > 0) And (r1 > 0)) And (r2 > r1)) Then r = r2
   If ((r2 > 0) And (r1 <= 0)) Then r = r2
   If ((r1 > 0) And (r2 <= 0)) Then r = r1
```

```
    s_bar = Application.Max(0, (t - Application.RoundDown(r, 0)))
    EG2 = 0
    For j = s_bar To t
       b_j1 = Exp(Application.GammaLn(a + b)) * Exp(Application.GammaLn(a + j)) * _
               Exp(Application.GammaLn(b + t - j))
       b_j2 = Exp(Application.GammaLn(a)) * Exp(Application.GammaLn(b)) * _
               Exp(Application.GammaLn(a + b + t))
       b_j3 = Application.Combin(t, j)
       b_j = b_j1 / b_j2 * b_j3
       EG2_1 = vw - ((vw + vl * q) * (b + t - j + 1) * (b + t - j) _
                 / ((a + b + t + 1) * (a + b + t)))
       EG2 = EG2 + (EG2_1 * b_j)
    Next j
    EG2 = Application.Max(EG2 * (m - t) / 2, 0)
    Range("E_G2").Select
    ActiveCell.Offset(t, 0).Value = EG2
    ActiveCell.Offset(t, 1).Value = s_bar
    GoTo next_t
Reject_t:
    Range("E_G2").Select
    ActiveCell.Offset(t, 0).Value = 0
    ActiveCell.Offset(t, 1).Value = 0
next_t:
 Next t
 Dim check_range As Range
 Dim optimal_t As Integer
 Dim t_counter As Integer
 Dim optimal_gain As Variant
 Sheets("Results").Select
 Range("B2").Activate
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Set check_range = Selection
 Range("max_gain").Select
 ActiveCell.Value = Application.Max(check_range)
 optimal_gain = Application.Max(check_range)
 Range("b2").Activate
 For t_counter = 1 To m + 1
    If ActiveCell.Value = optimal_gain Then
      optimal_t = ActiveCell.Offset(0, -1).Value
      GoTo loop_exit_4
    End If
    ActiveCell.Offset(1, 0).Activate
 Next t_counter
loop_exit_4:
 Range("t_star").Value = optimal_t
'
'This routine calculates the risks associated with accepting the
'system after testing.
'
 Dim t_star As Variant
 Dim s_bar_star As Variant
 Dim p_bar As Variant
 Dim p_bar1 As Variant
 Dim p_bar2 As Variant
```

```
t_star = Range("t_star").Value
r1_star = -b - 0.5 + 0.5 * (1 + ((4 * vw * (a + b + t_star + 1) * _
                    (a + b + t_star)) / (vw + vl * q))) ^ 0.5
r2_star = -b - 0.5 - 0.5 * (1 + ((4 * vw * (a + b + t + 1) * _
                    (a + b + t)) / (vw + vl * q))) ^ 0.5
If (((r2_star > 0) And (r1_star > 0)) And (r1_star > r2_star)) Then r_star = r1_star
If (((r2_star > 0) And (r1_star > 0)) And (r2_star > r1_star)) Then r_star = r2_star
If ((r2_star > 0) And (r1_star <= 0)) Then r_star = r2_star
If ((r1_star > 0) And (r2_star <= 0)) Then r_star = r1_star
s_bar_star = Application.Max(0, (t_star - Application.RoundDown(r_star, 0)))
Range("s_bar_star").Value = s_bar_star
m_t = m - t_star
N_mt = Application.RoundDown(m_t / 2, 0)
p_bar = 0
p_bar1 = 0
p_bar2 = 0
Sheets("Results").Select
Range("PM").Select
Range(ActiveCell, ActiveCell.End(xlDown)).Select
Selection.Clear
Range("k").Select
Range(ActiveCell, ActiveCell.End(xlDown)).Select
Selection.Clear
Range("PMD").Select
Range(ActiveCell, ActiveCell.End(xlDown)).Select
Selection.Clear
Sheets("Results").Select
Range("p_bar").Select
Selection.Clear
If ((t_star = 0) And Not (optimal_gain = 0)) Then
  GoTo NoTest
ElseIf (optimal_gain = 0) Then
  GoTo Reject
Else
For k = 0 To N_mt
PM1 = 0
PM2 = 0
   For j = s_bar_star To t_star
      PM1a = 0
      For n = 0 To k
         PM1a = PM1a + ((Application.Combin(t_star, j)) * _
                   (Application.Combin(N_mt, k)) * _
                   (Application.Combin(k, n)) * _
                   ((-1) ^ n) * _
                   Exp(Application.GammaLn(a + j)) * _
                   Exp(Application.GammaLn(2 * N_mt - 2 * k + 2 * n + b + t_star - j)) / _
                   Exp(Application.GammaLn(2 * N_mt - 2 * k + 2 * n + a + b + t_star)))
      Next n
   PM1 = PM1 + PM1a
   PM2 = PM2 + ((Application.Combin(t_star, j)) * _
            Exp(Application.GammaLn(b + t_star - j)) * _
            Exp(Application.GammaLn(a + j)) / _
            Exp(Application.GammaLn(a + b + t_star)))
   Next j
```

```
PM = PM1 / PM2
Sheets("Results").Select
Range("PM").Select
ActiveCell.Offset(k, 0).Activate
ActiveCell.Value = PM
Next k
Range("PMD").Select
ActiveCell.FormulaR1C1 = "=RC[-1]"
For i = 1 To N_mt
   ActiveCell.Offset(1, 0).Activate
   ActiveCell.FormulaR1C1 = "=RC[-1]+R[-1]C"
Next i
Range("k").Select
For i = 0 To N_mt
   ActiveCell.Value = i
   ActiveCell.Offset(1, 0).Activate
Next i
For j = s_bar_star To t_star
   p_bar1 = p_bar1 + ((1 - ((b + t_star - j + 1) * (b + t_star - j) / _
                  ((a + b + t_star + 1) * (a + b + t_star)))) * _
               Application.Combin(t_star, j) * _
               Exp(Application.GammaLn(a + j)) * _
               Exp(Application.GammaLn(b + t_star - j)))
   p_bar2 = p_bar2 + ((Application.Combin(t_star, j) * _
                Exp(Application.GammaLn(a + j)) * _
                Exp(Application.GammaLn(b + t_star - j))))
Next j
p_bar = (p_bar1 / p_bar2)
Range("p_bar").Select
ActiveCell.Value = p_bar
Range("p_bar_comment").Select
Selection.Clear
End If
GoTo Fini
Reject:
Range("t").Activate
For i = 0 To m
   ActiveCell.Offset(i, 0).Value = i
   ActiveCell.Offset(i, 1).Value = 0
Next i
Range("max_gain").Value = 0
Range("t_star").Select
Selection.Clear
MsgBox "Reject System"
Range("s_bar").Select
Range(ActiveCell, ActiveCell.End(xlDown)).Select
Selection.Clear
Range("s_bar_star").Value = "NA"
Sheets("Results").Select
Range("k").Select
Range(ActiveCell, ActiveCell.End(xlDown)).Select
Selection.Clear
Range("p_bar").Select
Selection.Clear
```

```
 Range("p_bar_comment").Select
 Selection.Clear
 Range("PMD").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 Range("PM").Select
 Range(ActiveCell, ActiveCell.End(xlDown)).Select
 Selection.Clear
 ActiveCell.Value = "REJECT"
 ActiveCell.Offset(0, 1).Activate
 ActiveCell.Value = "SYSTEM"
 GoTo Fini
NoTest:
 MsgBox "Optimal Gain Achieved with No Testing"
 Sheets("Results").Select
 Range("p_bar").Select
 Selection.Clear
 ActiveCell.Offset(0, 1).Value = "<-- Prior probability used since 0 tested"
 N_m = Application.RoundDown(m / 2, 0)
 Range("k").Select
 For k = 0 To N_m
 PM1 = 0
 PM2 = 0
 s_bar_star = 0
 t_star = 0
    For j = s_bar_star To t_star
       PM1a = 0
       For n = 0 To k
          PM1a = PM1a + ((Application.Combin(t_star, j)) * _
                   (Application.Combin(N_mt, k)) * _
                   (Application.Combin(k, n)) * _
                   ((-1) ^ n) * _
                   Exp(Application.GammaLn(a + j)) * _
                   Exp(Application.GammaLn(2 * N_mt - 2 * k + 2 * n + b + t_star - j)) / _
                   Exp(Application.GammaLn(2 * N_mt - 2 * k + 2 * n + a + b + t_star)))
       Next n
    PM1 = PM1 + PM1a
    PM2 = PM2 + ((Application.Combin(t_star, j)) * _
             Exp(Application.GammaLn(b + t_star - j)) * _
             Exp(Application.GammaLn(a + j)) / _
             Exp(Application.GammaLn(a + b + t_star)))
    Next j
 PM = PM1 / PM2
 Sheets("Results").Select
 Range("PM").Select
 ActiveCell.Offset(k, 0).Activate
 ActiveCell.Value = PM
 Next k
 Range("PMD").Select
 ActiveCell.FormulaR1C1 = "=RC[-1]"
 For i = 1 To N_mt
    ActiveCell.Offset(1, 0).Activate
    ActiveCell.FormulaR1C1 = "=RC[-1]+R[-1]C"
 Next i
```

```
 Range("k").Select
 For i = 0 To N_m
    ActiveCell.Value = i
    ActiveCell.Offset(1, 0).Activate
 Next i
 For j = s_bar_star To t_star
    p_bar1 = p_bar1 + ((1 - ((b + t_star - j + 1) * (b + t_star - j) / _
                    ((a + b + t_star + 1) * (a + b + t_star)))) * _
                  Application.Combin(t_star, j) * _
                  Exp(Application.GammaLn(a + j)) * _
                  Exp(Application.GammaLn(b + t_star - j)))
    p_bar2 = p_bar2 + ((Application.Combin(t_star, j) * _
                   Exp(Application.GammaLn(a + j)) * _
                   Exp(Application.GammaLn(b + t_star - j))))
 Next j
 p_bar = (p_bar1 / p_bar2)
 Range("p_bar").Select
 ActiveCell.Value = p_bar
Fini:
 Range("G5").Activate
End Sub
'
'This routine provides for the functions that occur on workbook opening.
'
Sub Auto_open()
 Call get_input
 ActiveWorkbook.Protect contents = True
End Sub
```

APPENDIX E. INSTALLATION INSTRUCTIONS

The installation diskette has the following three files:

1. G1.xls - the Excel workbook which contains the algorithm for analysis of the single weapon case.

2. G2.xls - the Excel workbook which contains the algorithm for analysis of the two-weapon salvo case.

3. Readme.txt - an ASCII text file which contains the text found in this appendix.

To use the analysis files, simply open them with Excel V7.0 or later and follow the dialog box prompts. Input parameters can be changed and the algorithm repeated by clicking the button on the worksheet marked "Press to Change Parameters." The workbooks open recommending a "read-only" format. This is done to protect the structure of the "Results" worksheet. The worksheets which contain the input dialog box and the source code are "hidden." The source code is also "protected" and cannot be altered. If one wishes to alter the code to develop the ideas of the software further, cut and paste the appropriate code into a different workbook and proceed.

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